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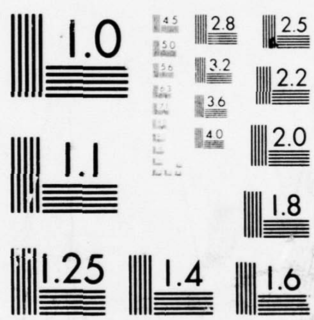

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6 GRAPHICAL REPRESENTATIONS AS A DISCIPLINE.

by

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DEPARTMENT OF MATHEMATICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS

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Herman Chernoff

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1. Introduction

Having spent the last few years on research removed from graphical representations, I welcome this symposium as an opportunity to review the field from a distance and to reflect on aspects that were less than perfectly clear a short time ago and are still somewhat hazy.

My first serious contact with this field occurred many years ago as an undergraduate student in an elementary statistics course where the students were obliged to present an india ink drawing of a histogram of data. My sloppy drawing was greeted with horror by the teaching assistant who insisted that it was unacceptable and had to be redone. That was bad enough but what really annoyed me was his reaction to my revised drawing. After studying the finished neat histogram which took hours of labor, he congratulated me, telling me that with a few drawings like this in my portfolio I would find it easy to gain employment as a statistician.

Considering the difficulty of finding jobs during those depression years, he evidently meant well. However, if the work of a statistician consisted of such non-intellectual labor, that profession was not for me.

Several years later I found myself drawn back to Statistics and eventually became a Professor of Statistics. My disrespect for drawing graphs remained and was reflected in part by the refusal to spend more than one lecture per course on histograms and ogives. Never never would I discuss pie charts nor diagrams with little men of various size. On the other hand I found myself counseling the students over and over again to draw diagrams and graphs and to "look" at the data.

What seems to have been involved in my behavior is a combination of standard psychological reactions. The great success of some graphical techniques is based on their simplicity and transparency. While their innovation required substantial intellectual insight, college students have grown up frequently exposed to these simple techniques and should not need to have them explained. Indeed the college student who doesn't understand graph paper will probably not be helped much by explanations. Another reaction is that what is easily understood seems trivial. Thus the teacher tends to underestimate the conceptual difficulties in aspects of his subject which have been fully integrated into his view of the world but which may be very novel to his students. In the process of explaining such sophisticated concepts as estimation and hypothesis testing, the teacher may forget that not everyone knows how graphs may be used for quick efficient calculations nor that naive students may not conceive of learning what a formula means by drawing a graph. The more natural one finds the use of graphs the less likely one is to feel that their usefulness requires explanation.

While everyone ought to know what a pie chart is, there are many intelligent people who do not tend naturally to think in terms of graphs and who find it relatively difficult to be innovative in the use of graphs and charts. For them it is of some importance to be exposed to a list of alternative methods of wide applicability, with detailed explanations of how to use these techniques as well as how not to use them.

For those more gifted with geometric insights, a few examples of innovative special purpose graphs can serve as an excellent introduction to the world of graphics. In any case there is no doubt that students of Statistics should be encouraged to use graphical techniques.

2. History

It has been widely noted, e.g., [6], that between 1940 and 1960 there had been a great decline in the attention paid by academic statisticians to graphical representation of data. Academic statisticians found the new analytical and conceptual aspects of their field more exciting. Lately, however, there has begun a substantial change in the attention paid to graphics as an intellectual discipline with important uses. This is reflected in part by several recent articles discussing Graphics and its history. Notable among these are those of Beniger and Robyn [2], Feinberg [6], Kruskal [9], and Royston [11].

One of the surprising facts arising from these articles is that the use of graphical techniques as an accepted practice dates back less than 200 years to the work of Crome and Playfair. Royston has suggested that Playfair, whose work was most influential, may have become acquainted with such techniques when he worked as a draftsman for Boulton and Watt.

It is possible that graphical techniques were commonly used by scientists and engineers before Playfair's work lead to their development in statistical application.

One may wonder why these techniques were not used more often in the past. Granted that Statistics was a new discipline, why weren't such techniques used elsewhere and why were they used so rarely in deference to tables in the nineteenth century. It is possible that the technology of printing charts in books was insufficiently well developed to encourage the formal use of charts in publications in spite of their numerous advantages.

Whatever was the role of technology in the past, it is clear that the development of computers has had a considerable impact on the reawakening interest in graphics. The computer has the capacity to produce vast amounts of answers. Thus the computer presents the problem of digesting great amounts of data, a problem for which graphical methods are well suited.

A second problem presented by the computer is that it encourages one to feed in data and receive the output of stereotyped package programs too passively. When regressions were tediously carried out by hand, discrepant data and inappropriate models were apt to be discerned in the laborious process of computing. An answer spit out by a reliable computer is likely to carry conviction unless it is ridiculously wrong. Thus it is especially important to build diagnostic devices into the program. Here again graphical techniques such as plotting residuals are potentially valuable.

Not only does the computer provide the needs for graphical methods, it also furnishes a tool to help produce the graphs with minimal labor (and sometimes with insufficient thought).

3. Classification

In Fienberg's recent paper [6], he refers to Schmid's

Handbook of Graphical Presentation [12] where suggestions for classifying charts and graphs are considered. Fi berg introduces an alternative classification of graphs in order to analyze the use of graphics in The Journal of the American Statistical As ation (JASA) and Biometrika from 1920 to 1975. For a historical study of trends of use, some such classification may be required. However, this one leaves much to be desired as a guide for developing a discipline of graphics.

To quote two of Fienberg's references to Schmid:

"The qualities and values of charts and graphs as compared with textual and tabular forms of presentation have been succinctly summarized by Calvin Schmid (1954) in his Handbook of Graphic Presentation:

1. In comparison with other types of presentation, well-designed charts are more effective in creating interest and in appealing to the attention of the reader.
2. Visual relationships, as portrayed by charts and graphs, are more clearly grasped and more easily remembered.
3. The use of charts and graphs saves time, since the essential meaning of large masses of statistical data can be visualized at a glance.
4. Charts and graphs can provide a comprehensive

picture of a problem that makes possible a more complete and better balanced understanding than could be derived from tabular or textual forms of presentation.

5. Charts and graphs can bring out hidden facts and relationships and can stimulate, as well as aid, analytical thinking and investigation."

"Schmid (1954) suggests that the basis for classifying charts and graphs must utilize one or more of the following criteria: (A) purpose, (B) circumstances of use, (C) type of comparison to be made, (D) form. Under purpose he lists the following three categories:

(1) Illustration,

(2) Analysis,

(3) Computation."

Then Fienberg presents an augmented list of six purposes developed by him and his graduate student Philip Chapman for the JASA - Biometrika study, the object of which was to determine whether the relative volume of usage of graphics had changed and whether there had been a shift in purpose to which graphs were being applied. Fienberg demonstrates a substantial decline in the use of graphics associated with data in these two statistical journals.

Fienberg's augmented list of purposes, presented with some explanation, are

"I. Graphs depicting theoretical relationships, such as probability density functions, contours of multivariate densities, values of risk functions, contours of multivariate densities, values of risk functions for different estimators, and theoretical descriptions of graphical methods.

II. Computational graphs and charts, used as substitutes for tables --- e.g. Fox's chart's, monograms, and especially charts with small detailed grid lines.

III. Non-numerical graphs and charts --- e.g. maps, certain skull diagrams, Venn diagrams, flow charts.

IV. Graphs intended to display data and results of analysis --- e.g. time series charts, histograms, results of Monte Carlo studies, scatter plots (even those with an accompanying regression line).

V. Plots and graphs with elements of both data display and analysis --- e.g. charts from older papers involving primitive forms of analysis; graphs of posterior distributions.

VI. Analytical graphs -- residual plots, half-normal and other probability plots where conclusions are drawn directly from graph, graphical methods of performing calculations, spectrum estimates from time series."

The development of a discipline of graphics seems to require a somewhat different approach in classification than that suggested by Fienberg. My feeling is that both graphical methods and potential applications should be measured on each of a list of relevant attributes. This list should contain the three purposes of Schmid and it is possible that a chart can serve 2 or 3 of these purposes simultaneously and that a particular application calls for more than one of these. The list should also contain the qualities and values mentioned by Schmid above. A list will be presented in Section 5 after a brief digression.

4. Representations of Multivariate Data

Before listing the attributes of charts we describe briefly 4 methods of presenting multivariate data graphically. Let $\underline{x} = (x_1, x_2, \ldots, x_k)$ be a k-dimensional vector to be described. The method of profiles represents $\underline{x}$ by a bar chart with the i-th bar at a height determined by x_i . A common variation consists of using a polygonal line connecting the points whose coordinates are (i, x_i) .

A second variation is the star in which the polygon is wrapped around a circle. To be more precise, the polygon connects the points whose polar coordinates are

$\rho_i = x_i$ and $\theta_i = 2\pi i/k$. Fienberg and others refer to Siegel, Goldwyn and Friedman [13] for using stars. I wish to thank R. J. K. Jacob for supporting my impression that this method had been used, at least informally, in the past by supplying a reference to Brinton [3,p.80] who refers to stars critically as a form of chart that should be banished to the scrap heap.

Andrews [1] introduced the representation using the Fourier Series Representation of $\underline{x}$ by plotting

$$f(\underline{x}) = x_1/\sqrt{2} + x_2 \sin t + x_3 \cos t + x_4 \sin 2t + \ldots$$

over the range $-\pi \leq t \leq \pi$.

Finally Chernoff [4] developed a computer program which draws a cartoon of a face determined by 18 parameters such as length of nose, curvature of the mouth, size of eyes, etc. If $k \leq 18$, one may adjoin to $\underline{x}$, 18-k specified numbers and use the resulting 18 component vector as the 18 parameters of a face to be drawn by the computer. The resulting face represents $\underline{x}$.

5. Attributes

We list many attributes for consideration in classifying charts. Each graphical method has many of these attributes in varying degrees. Each application requires various of these attributes to a greater or lessor extent. The key to the successful use of graphics should involve a matching of method and application in terms of the extents of the attributes required by the application and how well the method supplies these attributes.

The first three items refer to the purposes to be served. These are the following:

1. Illustrate or communicate.

   Here the object is to communicate to the audience information which has typically been analyzed and understood.

2. Analyze or comprehend.

   If data have not been well understood it is often useful to find a representation which permits the analyst to develop an understanding of what conclusions may be drawn and what relations and regularities exist. Graphs designed for analysis may be used frequently in "practice" but frequently do not appear in publications once their purpose of clarification has been served.

3. Compute.

   Some charts or graphs provide a means of doing relatively accurate computing of many desired quantities. Nomograms are especially designed for this purpose.

In dealing with multivariate data the stars and faces are potentially useful for analysis because the representations may present a gestalt which recalls psychologically meaningful objects or ideas. The Fourier series and the profile seem to be less valuable in producing an emotional response. It is plausible to expect a circular version of the Fourier series to be more effective in analysis just as stars are better than profiles. However the Fourier series representation has computational advantages. Squared distance in the vector space corresponds to squared integral and Andrews [1] has pointed out that good discriminant functions can be constructed from such graphs.

Faces can be used to communicate information to a limited extent <u>after some training</u>. Thus a wide smile can be made to correspond to good business or to a healthy patient and will be readily understood once the observer is given the <u>translation</u>. In this symposium Jacob [8] has described an innovative approach to using faces effectively for describing psychiatric patients. Goode [7] has developed a variation of faces consisting of a model of a football player whose measurements are determined by the qualities of a football team.

Other attributes of importance follow.

4. Impact.

If an important fact is to be communicated, it should be presented in a forceful fashion. A time series can have considerable impact in showing sudden changes or long term trends.

5. Mnemonic character.

When much information is to be communicated, it may be important to present some so that it will be well remembered. When complex data are to be analyzed the representation must be such that potentially important bits are not forgotten while observing others.

It is in this respect that the star is a great improvement over the profile for the profile makes it easy to confuse the index of the component which sticks out with the next one. The star improves in two respects. For small dimension $(k \leq 8)$, the direction of each vertex is easily remembered. The characteristic shapes that the star attains recall objects that help the memory. The latter explanation applies also to faces.

6. Attraction.

A chart addressed to a casual audience should attract attention. Playfair's charts were attractive aesthetically. The potential viewer may pass by an ugly or complicated looking chart.

7a. Accuracy (Precision).

For computing purposes the chart should enable one to make precise measurements easily.

7b. Accuracy (Lack of Distortion).

It is easy to draw charts which are precise but present a distorted impression. This is to be avoided in honest work designed to communicate.

8. Compactness.

For many purposes a table is a compact precise instrument for presenting knowledge to be used for reference. Sometimes a graph is even more compact and almost as precise.

9. Comprehensiveness.

Generally it is desirable that a representation be as comprehensive as possible. However the attempt to communicate too much information may distract one from the important facts or may repel the casual audience.

When one has not yet decided what is and is not important, it is valuable to have as much information as can be comprehended. Faces seem to be good for this in multivariate data examples.

10. Self Explanatory.

The incentive to study a chart depends greatly on how much explanation is required to use it.

Novel techniques face greater difficulties in being comprehended easily than do standard techniques. The self-explanatory quality of a method depends in part on the education of the viewer. A standard form such as a bar chart needs little explanation for most people. Brinton's criticism of stars [3] was based in part on his assessment of their difficulty in being understood. Incidentally his main interest in charts was as a tool for communication.

11. Time.

The speed in incorporating the information is one of the major advantages of many graphical techniques. Information to be used for reference may not demand speedy access.

12. Dimensionality.

The methods described in the preceeding section were designed for multidimensional data. Superficially it seems that the profile is extremely limited in the number of dimensions that can be handled effectively with it. I would estimate that in increasing order of ability are the Andrews Fourier Series, stars and faces. Each of these can be improved. The Andrews method, modified to wrap around a circle should improve its ability. The use of an inner and outer star should help the stars. A pair of faces effectively doubles the number of parameters covered and probably with little decrease in ability to detect changes or relations.

13. Theoretical vs. Data.

    Fienberg's classification [6] involved the distinction between charts based on data and those representing theoretical concepts.

14. Contrast or Sensitivity.

    For computing purposes the ability to detect slight differences is valuable. In analysis, the ability to detect outliers is important. These abilities require techniques which emphasize differences. Probability paper exploits the ability to detect non-linearities.

15. Ease of Application.

    Pencil and paper techniques are desirable when doing analysis. A technique such as faces would be impossible to apply without a computer. Even with the computer, it is desirable to have a set up that makes it possible to implement quickly, easily and flexibly, whatever technique is chosen.

16. Audience.

    The expected users should help determine the appropriate characteristics to be stressed in the representations to be used. It may make sense to classify the audience on several dimensions such as seriousness and training.

As research in graphics develops, it is to be expected that new attributes will be understood to be important while some of those listed above will diminish in importance. This is especially so since there is some interrelatedness among the items on the list.

6. Fundamental Tools

To make a discipline of graphical representations will require related research in technology and psychology plus innovation and the experimental testing of techniques.

The relevance of technology is certainly clear in the use of faces which would be impractical without a computer. It has been suggested earlier that the slow development and growth in use of graphical methods prior to and during the nineteenth century may have been due in part to the difficulty in publishing charts.

The relevance of psychology is even more basic. If we look at our list of attributes we see that many of these involve perceptions and psychological reactions and hence the tools to evoke these must be tailored to our psychological tendencies to respond.

One fundamental tool is the use of models. A model is an effective device for communicating when it is possible for the user to make an easy one to one transformation from the properties of the model to the importance characteristics of the real problem. The use of 3 dimensional models in construction probably dates back before writing. Picture writing and maps are examples. Goode's football player [7] is an adaptation of the idea of faces to create a model.

7. Attributes of Faces

In the last few years, the use of faces has attracted considerable attention. A certain amount of experimentation has taken place, e.g., [5]. Applications have ranged from craters on the moon [10] to Soviet Policy in Subsaharan Africa [14]. What are the graphical attributes well served by Faces? We shall go systematically through our list of attributes to see what one should expect.

First let me propose some conjectures about the underlying psychological phenomena which make faces potentially useful. I believe that we learn very early to study and react to real faces. Our library of responses to faces exhausts a huge part of our dictionary of emotions and ideas. We perceive the face as a gestalt and our built in computer is quick to pick out the relevant information and to filter out the noise when looking at a limited number of faces.

Our ability to distinguish between very similar faces involves a mechanism where the brain converts the face to a mental caricature on which it operates. Hence the cartoon caricatures of faces which resemble our mental caricature will probably be more effective as a graphical representation then either more realistic drawings or freakish caricatures far removed from our mental ones.

Using these conjectures and possibly others which have not been stated, and some experience with the use of faces let us go through our list of attributes.

Faces are of little use to illustrate or communicate unless the audience is specially trained in which case they can be of limited use. They are potentially very useful to analyze multivariate data to discern large complex relationships or to detect changes and similarities. They are almost useless for computation.

The emotional reactions of humans to faces triggers an impact. Our built in library of reactions to faces enhances the mnemonic character of this representation.

They can not be relied on for accuracy and are subject to distortion but they are comprehensive and self explanatory and remarkably quick to analyze. The dimensionality is high and the ability to detect contrasts excellent.

The ease of application depends on the software. It is potentially rather simple in an environment where data can easily be entered into the computer and interactive facilities can prompt the user.

Fineberg [6] carried out a comparison among Stars, Andrews Fourier Series and Faces on data involving 8 specimens of first lower premolar teeth from 9 animals. These were 3 humans, 4 gorillas and orangutangs, and 2 chimpanzees. These data were those used by Andrews [1] in his paper introducing his method. The data for each individual were the 8 principal components. In this study the faces suffered by the comparison.

However, the first 2 principal components carried almost all of the information. The fact that the method of principal components (basically a linear analysis) is so effective indicates that there is little to gain in analysis by using the more exploratory technique of faces or for that matter either of the other methods.

The use of the Andrews method is sensible for it permits one to calculate relevant distances and good linear discriminators. While the stars are effective in distinguishing the three groups of animals, a two-dimensional graph of the first two principal components is clearer and does better than any of these exotic techniques.

The effectiveness of the Faces was dependent on the two features used for the first two principal components. I suspect that the faces would do relatively better if the three methods were applied directly to the original data. Even so this is a problem when standard análysis is quite effective and none of these techniques is especially necessary.

8. Summary

The use of faces seems to be growing in popularity. It gives me great pleasure to participate in this symposium, a large part of which is devoted to the use of faces. It is clear that it is only one of a host of new innovative graphical methods that are being developed. As more methods are devised to use our growing technological capacities to cope with the problems of data representations, we must allocate some of our energy to develop graphics as a discipline.

This will require many inputs some of which are already flowing in greater quantities. In addition to innovative ideas, we need to develop further the technology of interactive statistical computing, we need to understand the psychology of perception better and we have much serious experimenting to do with graphical techniques. I hope that the classification concept proposed here will help clarify the issues to be settled by experimentation and help also in the proper design of these experiments.

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In recent years, partly as a result of the computer revolution, the use of graphical representations in Statistics is beginning to attract more attention. It seems appropriate to develop a discipline for such graphical representations. It is proposed that one way to develop such a discipline is to compile a list of functions and attributes on which each graphical method would be scaled and on which each potential application would also be scaled. For each application, the appropriate method could be selected by how well it matches the application in these attribute scales. Several representations of multivariate data including profiles, stars, faces and Andrews' Fourier Series are used as illustrations.

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